

MA111 - Engineering Mathematics - II
Problem Sheet - 6

Fourier Series

1. Find the Fourier series of the below periodic functions of period 2π .

(a) $f(x) = x^2$ ($0 < x < 2\pi$)

(b)

$$f(x) = \begin{cases} -4x & \text{if } -\pi < x < 0 \\ 4x & \text{if } 0 < x < \pi \end{cases}$$

2. Find the Fourier series of $f(x)$ of period p , where

(a)

$$f(x) = \begin{cases} 1+x & \text{if } -1 < x < 0 \\ 1-x & \text{if } 0 < x < 1, \end{cases} \quad p = 2$$

(b) $f(x) = \cos \pi x$ ($-\frac{1}{2} < x < \frac{1}{2}$), $p = 1$

3. Using the Fourier series of $f(x) = x^2$, $-1 < x < 1$, of period 2, show that $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \frac{\pi^2}{12}$.

4. Use the Fourier series of $f(x) = x^2$, $0 \leq x < 2\pi$, to find a series for $\frac{\pi^2}{6}$.

5. Use the Fourier series of

$$f(x) = \begin{cases} 0 & \text{if } -\pi \leq x \leq 0 \\ x^2 & \text{if } 0 < x < \pi, \end{cases}$$

to find a series for $\frac{\pi^2}{12}$.

6. Use the Fourier series of

$$f(x) = \begin{cases} 0 & \text{if } -1 < x < 0 \\ x & \text{if } 0 \leq x < 1, \end{cases}$$

to find a series for $\frac{\pi^2}{8}$.

7. Use the Fourier series of $f(x) = |\sin x|$, $-\pi \leq x \leq \pi$, to find a series for $\frac{\pi}{4}$.

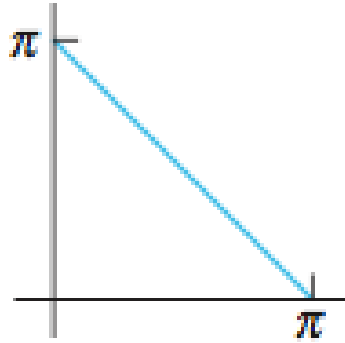
8. Is the given function even or odd? Find its Fourier series.

(a) $f(x) = 2x|x|$, $-1 < x < 1$

(b)

$$f(x) = \begin{cases} \pi e^{-x} & \text{if } -\pi < x < 0 \\ \pi e^x & \text{if } 0 < x < \pi, \end{cases}$$

9. Find the Fourier cosine series and the Fourier sine series of $f(x) = 2 - x, 0 < x < 2$.
10. Find the Fourier cosine series of $f(x) = \sin x, 0 < x < \pi$.
11. Find the Fourier cosine series and the Fourier sine series of



12. Find the sine series representation of $f(x) = e^{-x}, 0 < x < \pi$.